# Detection of Extreme Points of On-line Handwritten Scripts*

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## Abstract

On-line handwritten scripts consist of sequences of components that are pen tip traces from pen-down to pen-up positions. This paper describes a feature extraction procedure which detects the local extrema of curvature in individual components. The local extrema of curvature are useful features in accordance with the delta log-normal theory of handwriting generation. They can form a basis for script recognition.


## I Introduction

With the development of digitizing tablets and microcomputers, on-line handwriting recognition has become an area of active research since the 1960s [9]. On-line recognition refers to the recognition mode in which the machine recognizes the handwriting while the user writes on the surface of a digitizing tablet. The digitizing tablet captures the dynamic information about handwriting, such as traces of the pen tip from pen-down to pen-up positions, pen tip presure, writing speed, etc., all in real time.

In an on-line system, the recognition process usually consists of several steps: data pre-processing, feature extraction, and pattern classification. Feature extraction is a very crucial step, as the success of a recognition system is often attributed to a good feature extraction method. However, feature definition and extraction methods are very diverse in different recognition approaches[9], and some of the systems simply use data points which are roughly equally spaced in distance to characterize the handwriting traces[12].

In this paper, we present a procedure for on-line component analysis in accordance with the delta log-normal theory of handwriting generation[1, 4, 7]. The procedure detects the local extrema of curvature in individual components. The local extrema of curvature correspond to the maxima of angular velocity of pen tip movement. For simplicity and efficiency, this procedure does not use velocity information. Instead it uses the pen tip traces on the $x-y$ plane captured by a digitizing tablet for curvature analysis. Unlike the techniques for blob boundary analysis[8] and stroke corner detection[6] based on eight-neighbour chain code, this technique uses the boundary line segments to compute the sequence of changes in angles. The extrema of curvature are detected based on this information.

The remainder of this paper is divided into four sections. Section II describes the details of this procedure. Section III presents the experimental results. Section IV discusses possible applications, and finally, section V gives our conclusions.

## II Detection Algorithm

### II.1 Handwriting component

In the current context, a handwriting component[10, 13] refers to the trace of the pen tip from pen-down to pen-up positions captured by the digitizing tablet. It can therefore be described as a sequence of consecutive points on the $x-y$ plane:

$$C = p_1^{(o)} p_2^{(o)} \cdots p_N^{(o)} \tag{1}$$

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where $p_n^{(o)} = (x_n^{(0)}, y_n^{(0)})$, $p_n^{(0)} \neq p_{n+1}^{(0)}$, $1 \leq n \leq N-1$, $p_1^{(o)}$ is the pen-down point, $p_N^{(o)}$ is the pen-up point, and $N$ is the number of sampled points in the trace. Based on this description, an on-line handwritten script can be expressed as a sequence of components, and can be characterized by features of the components. Among different features, the local extrema of curvature of individual components are perceptible by our human eyes and are useful in component segmentation and shape analysis.

## II.2 Curvature approximation

In differential calculus, the curvature $c$ at a point $p$ on a continuous plane curve $C$ is defined as

$$c = \lim_{\Delta s \to 0} \frac{\Delta \alpha}{\Delta s} \tag{2}$$

where $s$ is the distance to the point $p$ along the curve and $\Delta\alpha$ is the change in the angles of the tangents to the curve at distance $s$ and $s + \Delta s$, respectively.

Since the sign of curvature is related to the curve direction, one can define convex and concave curvature to reflect this relationship:

- $c$ is convex if and only if $c < 0$.
- $c$ is concave if and only if $0 < c$.

In the light of the above definitions, the direction of the tangent line always turns clockwise with a convex curvature; whilest the direction of the tangent line always turns counterclockwise with a concave curvature.

In practical cases, it is difficult to calculate the above limit when the analytical format of the curve is not available. However, By regarding the line segments formed by consecutively sampled points as the tangent lines, then interpolating data points to form the pixel unit along a component such that

$$C = p_1 p_2 \cdots p_L \tag{3}$$

where $p_1 = p_1^{(0)}$, $p_L = p_N^{(0)}$, $p_{l+1} \neq p_l$, satisfying

$$\begin{array}{l} |x_{l+1} - x_l| = 0 \quad or \quad 1 \\ |y_{l+1} - y_l| = 0 \quad or \quad 1 \end{array} \tag{4}$$

one can easily obtain the sequence of angles from point to point:

$$A = \alpha_1 \alpha_2 \cdots \alpha_L \tag{5}$$

where $\alpha_l$ ($0° \leq |\alpha_l| \leq 180°$) is the angle of the tangent line at $p_l$, $1 \leq l \leq L$, which is determined by a corresponding line segment from $p_{n-1}^{(0)}$ to $p_n^{(0)}$:

$$\alpha_l = tan^{-1} \frac{y_n^{(0)} - y_{n-1}^{(0)}}{x_n^{(0)} - x_{n-1}^{(0)}} \tag{6}$$

In the light of $A$, one can also easily obtain the sequence of changes in angles:

$$\Delta A = \Delta\alpha_1 \Delta\alpha_2 \cdots \Delta\alpha_L \tag{7}$$

where

$$\Delta\alpha_l = \begin{cases} \Delta\alpha_l - \Delta\alpha_{l-1} + 360° & \Delta\alpha_l - \Delta\alpha_{l-1} < -180° \\ \Delta\alpha_l - \Delta\alpha_{l-1} & -180° < \Delta\alpha_l - \Delta\alpha_{l-1} < 180° \\ \Delta\alpha_l - \Delta\alpha_{l-1} - 360° & 180° < \Delta\alpha_l - \Delta\alpha_{l-1} \\ \pm 180° & other \quad cases \end{cases}$$

This condition guarantees that $\Delta\alpha \in [-180°, 180°]$. In the extreme cases that $\Delta\alpha_l - \Delta\alpha_{l-1} = \pm 180°$, $\Delta\alpha_l$ has the same sign as that of $\Delta\alpha_{l-1}$, since the original signal is supposed to be continuous.

It is obvious that the above sequence contains the curvature signal mixed with digitization and quantization noise. By convolving the sequence with a Gaussian filter in the spatial domain, the influence of the noise can be suppressed.

## II.3 Local extrema of curvature

A local extreme of curvature can be defined within an arbitrary small neighbourhood in the continuous case. In the discrete case, things are little different since there exists digitization-quantization noise. We define a local extreme of curvature based on signal intensity within a discrete neighbourhood.

Let $\Delta A^*$ be the filtered version of $\Delta A$, we define the following measures:

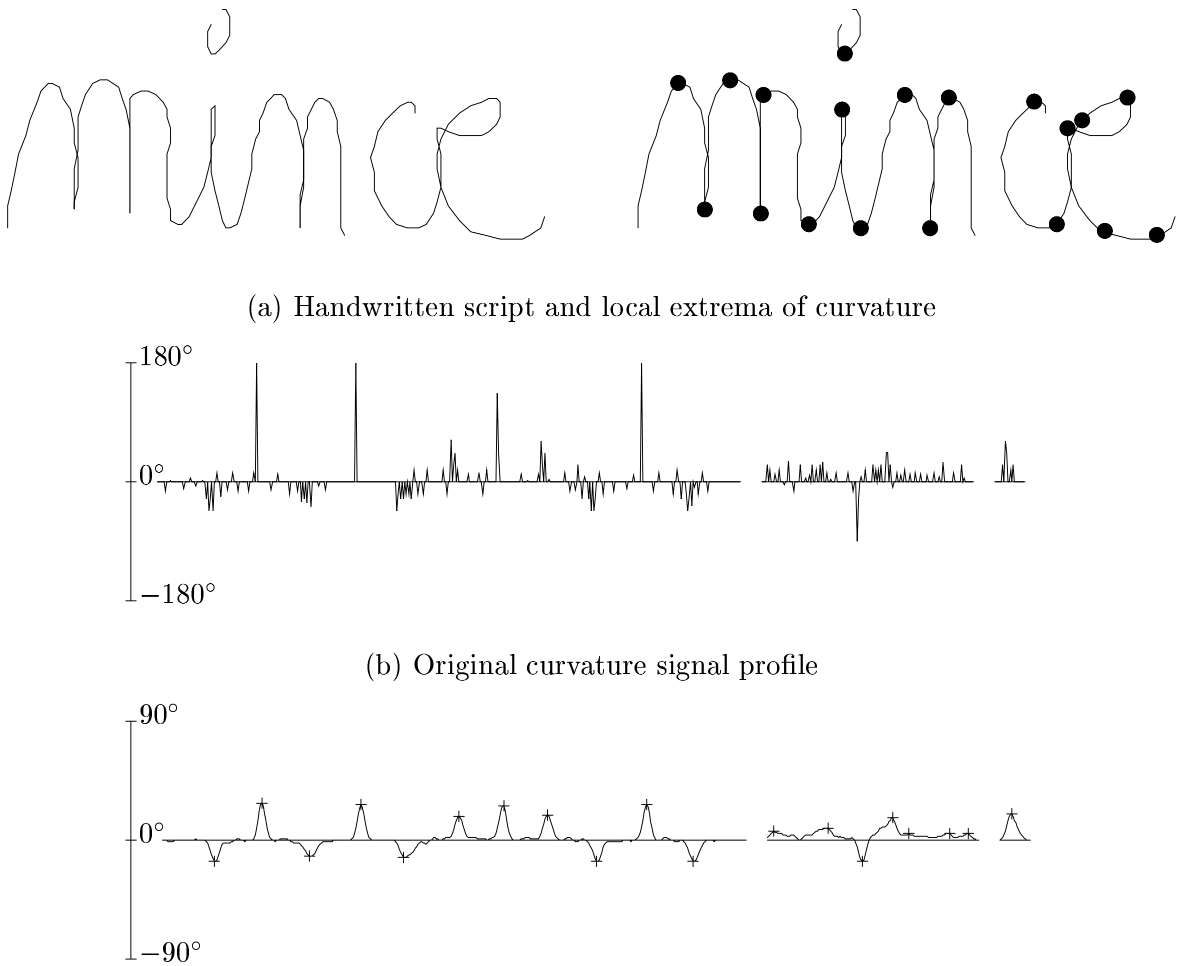
1. signal intensity

$$I = \sqrt{\frac{1}{L} \sum_{l=1}^{L} \Delta\alpha_l^* \times \Delta\alpha_l^*} \tag{8}$$

2. first order zero crossing point

   $p_l$ is a first order zero crossing point if and only if

$$\begin{array}{l} 0 \leq \Delta\alpha_l^* - \Delta\alpha_{l-1}^* \\ \Delta\alpha_{l+1}^* - \Delta\alpha_l^* < 0 \end{array} \quad or \quad \begin{array}{l} \Delta\alpha_l^* - \Delta\alpha_{l-1}^* < 0 \\ 0 \leq \Delta\alpha_{l+1}^* - \Delta\alpha_l^* \end{array} \tag{9}$$




(a) Handwritten script and local extrema of curvature

(b) Original curvature signal profile

(c) Filtered curvature signal profile

Figure 1: Example of extreme point detection

3. threshold of signal intensity

$$T = k_S I + k_L \tag{10}$$

where $k_S$ is the scaling coefficient and $k_L$ gives the lowest threshold value.

4. size of discrete neighbourhood

$$R = \begin{cases} r_1 & \frac{L}{M} < r_1 \\ \frac{L}{M} & r_1 \leq \frac{L}{M} \leq r_2 \\ r_2 & r_2 \leq \frac{L}{M} \end{cases} \tag{11}$$

where $L$ is the length of the sequence, $M$ is the number of first order zero crossing points in $\Delta A^*$, $r_1$ and $r_2$ are the lower-bound and upper-bound size of a neighbourhood.

Using the above measures, we are now in a position to define the points of local extrema of curvature along a discrete component:

- If $p_l$ is a first order zero crossing point and $\Delta\alpha_l^*$ is a local minimum within a discrete neighbourhood such that $\Delta\alpha_l^* \leq -T$ and $\Delta\alpha_l^* \leq \Delta\alpha_{l\pm r}^*$, $1 \leq r \leq R$, then $p_l$ is a point of local minimum of curvature.
- If $p_l$ is a first order zero crossing point and $\Delta\alpha_l^*$ is a local maximum within a discrete neighbourhood such that $T \leq \Delta\alpha_l^*$ and $\Delta\alpha_{l\pm r}^* \leq \Delta\alpha_l^*$, $1 \leq r \leq R$, then $p_l$ is a point of local maximum of curvature.

All the above points constitute the points of local extrema of curvature in a component.

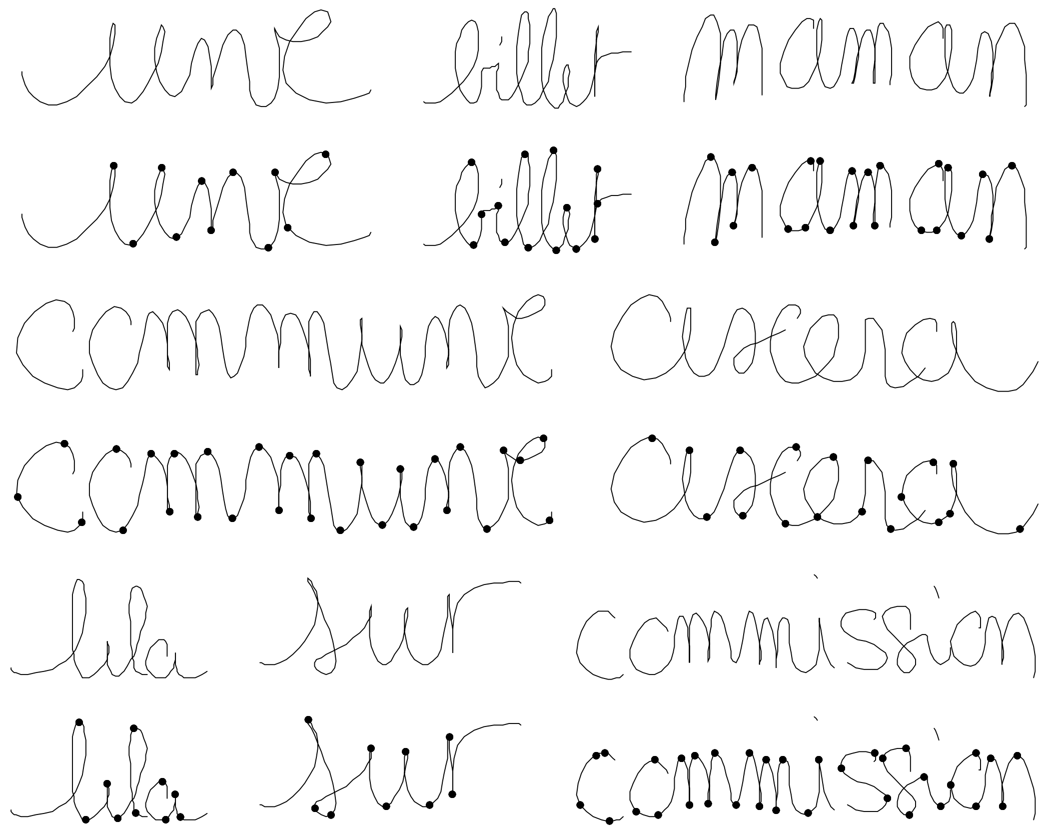


Figure 2: Results of extreme point detection (1)

## III Experimental Results

So far we have described our procedure for detection of extreme points in individual handwriting components. The results of our experiment using this technique are presented and discussed in this section.

The database used in our experiment contains 55 scripts (words) written by different people. Before applying this technique, each of these scripts was scaled as 64 pixel height with the width kept proportional to its original size. For each component in all the above cases, $\Delta A$, the sequence of changes in angles was iteratively filtered twice using the following equations:

$$\Delta\alpha_l^* = \frac{1}{W}\sum_{s=l-8}^{l+8} w_s \Delta\alpha_s \tag{12}$$

where

$$w_s = e^{-[0.375(s-l)]^2} \tag{13}$$

$$W = \sum_{s=l-8}^{l+8} w_s \tag{14}$$

and the coefficients and bounds related to the threshold and the size of neighbourhood are set as $k_S = 0.25$, $K_L = 4$, $r_1 = 4$, and $r_2 = 8$. Some of the detection results using this technique with the above parameters are shown in figures 1, 2, and 3.

The script in figure 1 (a) consist of 3 components with the points of local extrema of curvature marked by small circles. The original curvature signal profile and the filetered curvature signal profile of this script are displayed in figure 1 (b) and (c), respectively. The original signal is quite noisy but the filtered signal has clear peaks and valleys. From figure 1 (c) one can clearly see that the first component has 11 extrema detected, the second one has 7, and the third one has only 1 extreme (all these extrema are marked by crosses).

Figures 2 and 3 show more detection results, with line of scripts and their extrema alternatively displayed. From these figures one can see that the location of the extreme points detected is accurate, even though the constitute components are in a various scale compared with the fixed 64 script height. For brevity, the signal profiles related to these scripts are omitted.

# IV Possible Applications

The extreme point detection described in this paper may have the following applications in on-line handwriting recognition:

- Component segmentation for structural analysis

  A model-based segmentation framework for handwriting processing has been proposed and simplified by Plamondon[1, 7]. In this framework the pen tip velocity is modeled as a vector controlled by two synergies. Based on this model, handwritten components are segmented into strokes, which are defined with respect to the impulse responses of the agonist and antagonist generators. The theory predict that complex velocity patterns emerge from the vectorial summation of basic stroke velocity vectors, each one having a module described by a delta lognormal law [2]. Due to superimposition processes, strokes are hidden in the trajectory and maximum curvature points can be used to recover them partially. In a practical sense, a stroke can thus be considered as portion of a component characterized by two consecutive minima of the curvilinear velocity, or by two consecutive maxima of the angular velocity. The sets of the strokes then can be used in allograph modeling and segmentation[5].

  The detection procedure described in this paper is in accordance with the above framework, although it does not use the velocity information directly. The extrema of curvature detected in a component correspond to the maxima of angular velocity in generating that component.

- Sequences of feature points for elastic matching

  The extrema of curvature detected in a component together with pen-down/pen-up and middle points between the extrema can form sequences of feature points which contain various information about curvature, location, and direction from point to point. These feature sequences can be used for pattern classification by dynamic time warping, either at character level[3] or word level.

- Sequence of observation symbols for HMM approach

  The sequences of feature points can be converted into a sequence of observation symbols through vector quantization. Similar to speech recognition[11], the sequences of observation symbols of a character or word then can be used either in training the Hidden Marcov Model of a class, or in recognizing the handwriting with the well trained HMM models.

# V Conclusion

In this paper we have presented a feature extraction procedure in accordance with the delta log-normal theory of handwriting generation[1]. This procedure detects the local extrema of curvature in individual components, which correspond to the maxima of angular velocity in pen tip movement. We have shown, through theoretical analysis and experimental results, that this detection scheme is efficient in computation and accurate in extreme point location. Possible applications using the above features in on-line recognition have also been suggested.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# References


[1] R. Plamondon. A delta lognormal model for handwriting generation. Proccedings of 7th Biennal Conference of the International Graphonomics Society, pp.126-127, 1995.

[2] R. Plamondon. A kinematic theory of rapid human movements Part I. Movement representation and generation. Biological Cybernetics, No.72, pp.295-307, 1995.

[3] X. Li and D.Y. Yeung. On-line handwritten alphanumeric character recognition using feature sequences. Proccedings of the 3rd International Computer Science Conference, pp.197-204, 1995

[4] R. Plamondon. Handwriting generation: the delta log-normal theory. *Proceedings of the Fourth International Workshop on Frontiers in Handwriting Recognition*, pp.1–10, 1994.

[5] M. Parizeau and R. Plamondon. Allograph adjacency constraints for cursive script recognition.

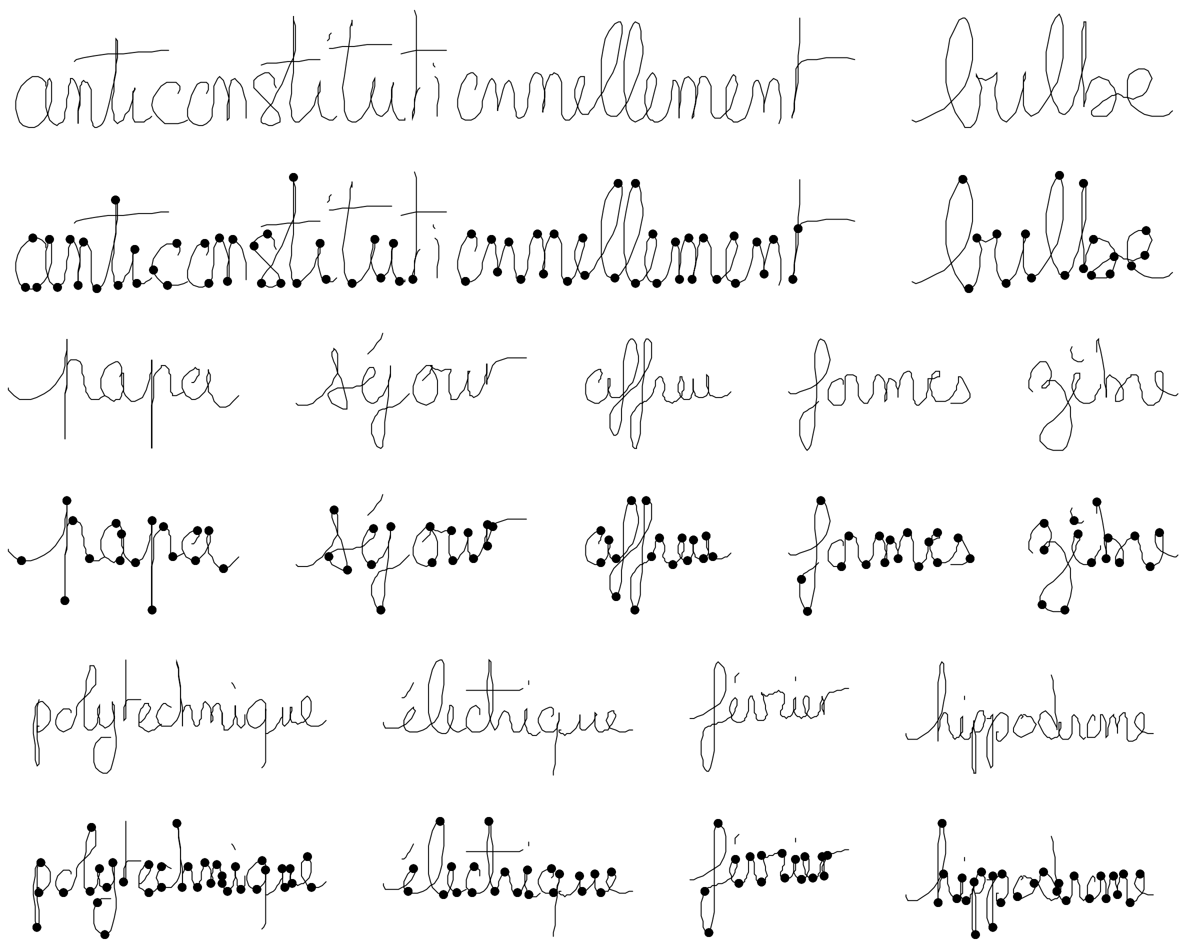

Figure 3: Results of extreme point detection (2)


Proccedings of the Third International Workshop on Frontiers in Handwriting Recognition, pp.252–261, 1993.

[6] X. Li and N.S. Hall. Corner detection and shape classification of on-line handprinted Kanji strokes. *Pattern Recognition*, vol.26, no.9, pp.1315–1334, 1993.

[7] R. Plamondon. A Model-based segmentation framework for computer processing of handwriting. *Proceedings of the Eleventh IAPR International Conference on Pattern Recognition*, pp.303–307, 1992.

[8] X. Li, M.A. Ireton and C.S. Xydeas. Detection of the extreme points of Closed contours. IEE Proccedings-I Communications, Speech and Vision, Vol.139, No.2, pp.198–205, 1992.

[9] C.C. Tappert, C.Y. Suen and T. Wakahara. The state of the art in on-line handwriting recognition. IEEE Trans. on Pattern Analysis and Machine Intelligence, Vol.PAMI-12, No.8, pp.787–808, 1990.

[10] R. Plamondon and F. J. Maasse. An evaluation of motor models of handwriting. IEEE Trans. System, Man and Cybernetics, Vol.19, pp.1060-1072, 1989.

[11] L. R. Rabiner. A tutorial on hidden Marcov models and selected applications in speech recognition. IEEE Stochastic Approaches, pp.267-296, 1989.

[12] C.C. Tappert. Cursive script recognition by elastic matching. IBM J. RES. DEVELOP, Vol.26, No.6, pp.765–770, 1982.

[13] G. Lorette. Méthodes de traitements automatiques d'images de points et de lignes en reconnaissance de formes. Thèse de Doctorat d'état, Université de Paris XII, 1983.