

# Optimal Scheduling for Smart Charging of Electric Vehicles using Dynamic Programming

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**Abstract.** We are proposing a formulation of the smart charging problem that can be solved by dynamic programming. It allows the optimal charging schedule of EVs to be determined in order to minimize the cost considering the different driving patterns of each car owner as well as the electricity prices varying according to supply and demand. Conclusive experiments are made through simulations, relying upon a database storing the history of the real use of vehicles over several months and an hourly electricity price.

**Keywords:** Smart charging, electric vehicles, sequential decision-making, dynamic programming, smart grid

## 1 Introduction

Widespread adoption of Electric Vehicles (EVs) may lead to a dramatic increase in electricity demand at peak time, considering that common usage patterns may lead to many vehicles being charged simultaneously (e.g., at supper time). There is thus a particular interest in integrating Demand-Side Management (DSM) into EV charging [1], which would provide financial incentives to EV users using smarter charging approaches that are avoiding demand peaks and as such flattening of the load curve.

Our paper is proposing a method to determine the optimal EVs charging schedule in order to minimize cost owners' charging cost considering driving patterns of each EV [2]. We formulate this as a sequential decisions problem, where an action (charge or standby) should be taken at each time step when the vehicle is connected. Furthermore, when the vehicle is not connected, a penalty is calculated if there is not enough energy available to complete the trip.

Similarly to the method that we propose and describe in the following sections, several other authors have proposed to use Dynamic Programming (DP) to determine the optimal charging schedule of an EV [3,4,5]. However, these optimization approaches are assuming predefined connection times and charging needs but none have captured the dependence between the charging decisions and the real energy consumption, which can be impractical and lead to inefficient schedules that are not based on actual EV user requirements. Our approach

is evaluated and compared with baseline approaches through computer simulations, having real databases of measured data collected from a fleet of cars, and the data on the price of gasoline as well as the hourly Ontario energy price.

The remainder of the paper is organized as follows. In Sec. 2, the mathematical model of the smart charging problem is provided. The modelling of this optimization problem as a Markov Decision Process (MDP) and its resolution with DP is provided in Sec. 3. Experiments with real-life datasets are presented in Sec. 4, demonstrating the effectiveness of the proposed approach, before concluding the paper in Sec. 5.

## 2 Optimization Problem

We define the optimal charging schedule of an EV as the best sequence of decisions (i.e., to charge the vehicle or to leave it on standby mode) when the EV is plugged in, in order to minimize the overall cost while satisfying the needs of the vehicle's owner. The charging decisions, which should be made at each time interval (e.g., 15-minute period), are based on the electricity price information, the State of the Charge (SoC) of the batteries, and the energy consumed by each vehicle.

The mathematical formulation is divided into two parts corresponding to two states, namely when the car is plugged in ( $S_p$ ) and when it is not ( $S_u$ ). When the vehicle is plugged in, the cost function is given as:

$$S_p(t) = C_{el}(t) \frac{E_{ch}(SoC(t))}{\eta}, \quad (1)$$

where:

- $C_{el}(t)$  is the electricity price [\$/kWh] at time  $t$ ;
- $E_{ch}(SoC(t))$  is the energy [kWh] supplied to the battery when charging is conducted during a fixed time interval;
- $SoC(t)$  is the charging fraction of the nominal battery capacity at the beginning of time period  $t$ ;
- $\eta \in [0, 1]$  is the charger efficiency.

When the vehicle is not plugged in, there is no decision to make, but it is necessary to compute the new SoC at the end of the interval. Moreover, a cost can be induced for driving with a depleted battery, which corresponds to the following cost with a Plug-in Hybrid Electric Vehicle (PHEV) (i.e., driving on the gas engine):

$$S_u(t) = C_{fuel}(t) \cdot \max(F_c(SoC(t)), 0), \quad (2)$$

where  $C_{fuel}(t)$  is the gasoline price [\$/l] at time  $t$  and  $F_c(SoC(t))$  is the fuel consumed in litres by the vehicle in a time interval corresponding to complete battery depletion, with an  $SoC(t)$  energy level at the beginning of the time interval.

Combining Eq. 1 and Eq. 2 yields the following optimization objective:

$$\min_{\{a(t)\}_{t=1}^T} \sum_{t=1}^T [a(t) \cdot z(t) \cdot S_p(t) + (1 - z(t)) \cdot S_u(t)], \quad (3)$$

where  $a(t)$  is the decision variable on whether the vehicle is being charged (1) or on standby (0) at time  $t$  and  $z(t)$  returns a Boolean value indicating whether the vehicle is plugged in (1) or not (0) to a charging station at the time interval  $t$ . This objective is subject to technical restrictions of the battery, which indicates that the power used to charge an EV ( $E_{ch}(SoC(t))/\eta$ ) cannot exceed the capacity available in the battery ( $E_{nom}(1 - SoC(t))$ ). The cost  $S_u(t)$  is null except when the battery is depleted, in which case the vehicle would incur a penalty. For PHEV, the gas engine acts as a backup when the batteries are depleted. In the case of a Battery Electric Vehicle (BEV),  $C_{fuel}(t)$  can be modified by adding a stronger penalty associated with running out of energy.

### 3 Resolution Through Dynamic Programming

In order to find solutions to the optimization problem given by Eq. 3, we propose to use a recursive formulation of the problem that can be solved through DP [6]. Let us define the  $Q(s(t), a)$  function as holding values in a three-dimensional array, with values evaluated recursively backwards (i.e.,  $t = T, T-1, \dots, 1$ ), over all the states  $s \in \mathcal{S}$  and actions  $a \in \mathcal{A}$ .  $Q(s(t), a)$  is computed as:

$$Q(s(t), a) = \begin{cases} s(t) \cdot \overline{C_{el}} & \text{if } t = T \\ r(s(t), a) + \max_{a \in \mathcal{A}} Q(s(t+1), a) & \text{otherwise} \end{cases}. \quad (4)$$

The following elements are composing the function:

- $\overline{C_{el}}$  is the average electricity price over  $t = 1, \dots, T$ ;
- $s \in \mathcal{S}$  is a discretized value of the SoC over  $B$  bins:

$$s(t) = \frac{\lfloor SoC(t) \cdot B \rfloor + 0.5}{B}; \quad (5)$$

- $a \in \mathcal{A}$  is an action, with  $a = 1$  corresponding to charging and  $a = 0$  to standby mode;
- $r(s(t), a)$  is the immediate cost function (reward), in our case it is equal to the value of the energy actually transferred to the battery or the penalty cost associated with driving on the gas engine when the batteries are depleted:

$$r(s(t), a) = \begin{cases} 0 & \text{if } z(t) = 1 \text{ and } a = 0 \\ -C_{el}(t) \cdot \frac{E_{ch}(SoC(t))}{\eta} & \text{if } z(t) = 1 \text{ and } a = 1 \\ -C_{fuel}(t) \cdot F_c(SoC(t)) & \text{if } z(t) = 0 \end{cases}. \quad (6)$$

Once the values of the 3D array composing the  $Q(s(t), a)$  function are computed by DP, the decision in state  $s(t)$  is made according to the best  $Q$ -value:

$$a^*(t) = \operatorname{argmax}_{a \in \mathcal{A}} Q(s(t), a). \quad (7)$$

## 4 Experiments

For our experiments, we assumed that we are using a plug-in hybrid EV similar to the Chevrolet Volt 2013. We completed the specifications with the parameters of Panasonic Lithium-ion NRC18650 batteries given that full specifications of the Volt batteries are not publicly available. It should be stressed that the optimization procedure proposed in this paper is independent from this battery model. Any models, as complex as they can be, can be used as long as they allow the computation of the new SoC resulting from charging for a given duration.

We use a simple method to calculate  $F_c$  considering the  $MPG_e$  miles per gallon gasoline equivalent of the vehicle ( $1 \text{ } MPG_e \approx 0.0470 \text{ [km/kWh]}$ ) and the distance travelled during a time interval. To calculate  $E_{ch}$  we used a simple battery model based on an equivalent electric circuit containing a voltage source in series with a resistor. Given space and relevance concerns, the details of this charging battery model are omitted.

The experiments rely on a database on the use of conventional vehicles collected for a fleet of cars in the city of Winnipeg<sup>1</sup>. We also used the hourly Ontario energy price<sup>2</sup>, that is the hourly price that is charged to local distribution companies in Ontario and the price of regular unleaded gasoline<sup>3</sup> provided by the Ontario Ministry of Energy, which publishes the weekly average prices paid by the consumers across the province.

### 4.1 Experiment Parameters

Two experimental parameters need to be defined: the number of bins ( $B$ ) used to discretize the SoC in DP and the time slot duration for decision-making ( $\Delta_t$ ). Fig. 1 shows a direct relation between the length of the time interval and the operational charging cost (Eq. 4), analyzed in two months of summer and two months of winter.

Similarly, the effectiveness of the  $\Delta_t$  parameter for intervals of 30 and 60 minutes is significantly hindered in comparison to the effectiveness of intervals of 15 minutes. This is expected as a small time interval provides greater flexibility to the decision-making process. With respect to  $B$ , a discretization of 20 bins appears to be a satisfactory tradeoff, as this number of bins reflects state changes of an order of accuracy of 5%, considered as sufficient given the complexity of the problem. Note that a finer time interval and a high  $B$  parameter also imply a substantial increase of the model complexity and the computational burden.

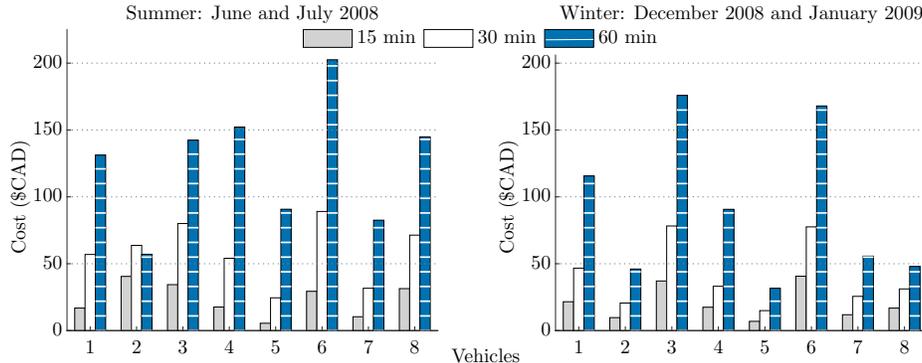
### 4.2 Results

From this point onwards, a response interval of  $\Delta t = 15$  minutes (i.e. 96 time slots per day) and two different periods, that is August 2008 (Summer) and February 2009 (Winter) are used in the experiments.

<sup>1</sup> <http://hdl.handle.net/1993/8131>

<sup>2</sup> <http://www.ieso.ca/power-data>

<sup>3</sup> <http://www.energy.gov.on.ca/en/fuel-prices>



**Fig. 1.** Average cost of eight vehicles for different numbers of bins by  $\Delta t$  for the summer and winter datasets.

Table 1 reports the cost of each vehicle for four situations: 1) when it depends exclusively on gasoline (GAS), 2) by using DP as the optimal charging strategy proposed, 3) by using the Random Decisions (RD) policy which consists in randomly selecting the action (charge or standby) and 4) by using a simple Always Charge (AC) strategy in which the EV users charge their vehicle whenever possible as long as its battery is not full, with no considerations for energy needs or electricity energy price.

It appears clear that the proposed DP is able to significantly enhance the results in comparison with the simple RD and AC strategies, thereby reaching 90% and 89% median gains in summer and winter, respectively. These gains come from having a charging schedule that is taking into account the electricity price and energy required for making the next trips.

## 5 Conclusions

This paper proposes a cost-effective solution to the problem of smart charging considering driving patterns. The results show that smart charging can significantly reduce the cost of charging EVs in a context of variable electricity pricing, as is the case in Ontario, Alberta, California, etc. While it is true that the proposed DP approach requires knowing in advance the energy price and car usage for a given period of time, the decisions obtained by DP using a MDP framework can be used as desired output values to infer decision models through some supervised machine learning methods over several states composed by ancillary variables (e.g., electricity price, time of day, weekdays, SoC).

This decision-making problem and associated resolution approach, which considers real activities at non-connection times to evaluate the decisions taken in connection times, can help to personalize the driving patterns of vehicles to make wise decisions based on real requirements of vehicles. Moreover, the method can

**Table 1.** Costs and gain ( $\$GAS - \$Model / \$GAS$ ) regarding gasoline-only vehicles (GAS) for the Dynamic Programming (DP), Random Decisions (RD) and Always Charge (AC) models in summer and winter test datasets (August 2008 and February 2009, respectively), for the 8 vehicles evaluated.

	Summer				Winter			
	GAS \$	DP \$ (%)	RD \$ (%)	AC \$ (%)	GAS \$	DP \$ (%)	RD \$ (%)	AC \$ (%)
1	131.2	34.0 (74)	93.7 (29)	91.0 (31)	65.1	7.9 (88)	30.1 (54)	29.1 (55)
2	93.0	22.1 (76)	57.1 (39)	57.4 (38)	45.5	5.4 (88)	24.8 (45)	22.2 (51)
3	187.4	15.7 (92)	60.4 (68)	60.6 (68)	66.5	7.4 (89)	28.7 (57)	27.2 (59)
4	230.8	66.8 (71)	184.1 (20)	165.6 (28)	58.8	6.3 (89)	25.2 (57)	24.6 (58)
5	81.0	3.2 (96)	20.7 (74)	21.4 (74)	40.5	4.2 (90)	18.8 (53)	18.3 (55)
6	156.8	17.3 (89)	63.0 (60)	56.5 (64)	108.3	14.2 (87)	47.4 (56)	44.4 (59)
7	20.8	0.4 (98)	5.8 (72)	6.2 (70)	61.8	6.4 (90)	27.2 (56)	25.8 (58)
8	32.0	2.3 (93)	8.3 (74)	9.6 (70)	52.8	7.2 (86)	24.7 (53)	23.0 (56)
<b>Mean gain</b>		<b>86%</b>	<b>54%</b>	<b>55%</b>		<b>88%</b>	<b>54%</b>	<b>56%</b>
<b>Median gain</b>		<b>90%</b>	<b>64%</b>	<b>66%</b>		<b>89%</b>	<b>55%</b>	<b>57%</b>

be used to label a dataset on smart charging, which in turn would allow training of a classifier on a variety of auxiliary variables, and this classifier would run in real time to decide whether a plugged-in vehicle should be charged or not.

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### References

1. Mou, Y., Xing, H., Lin, Z., Fu, M.: Decentralized optimal demand-side management for PHEV charging in a smart grid. *IEEE Trans. on Smart Grid* **6**(2) (2015)
2. Jia, L., Tong, L.: Dynamic pricing and distributed energy management for demand response. *IEEE Trans. on Smart Grid* **7**(2) (2016)
3. Li, Z., Khaligh, A., Sabbaghi, N.: Minimum charging-cost tracking based optimization algorithm with dynamic programming technique for plug-in hybrid electric vehicles. In: *Proc. of the Vehicle Power and Propulsion Conference (VPPC)*. (2011)
4. Rotering, N., Ilic, M.: Optimal charge control of plug-in hybrid electric vehicles in deregulated electricity markets. *IEEE Trans. on Power Systems* **26**(3) (2011)
5. Sarabi, S., Kefsi, L.: Electric vehicle charging strategy based on a dynamic programming algorithm. In: *Proc. of the Intl Conference on Intelligent Energy and Power Systems (IEPS)*. (2014)
6. Bellman, R.: *Dynamic Programming*. Dover Publications (1957)